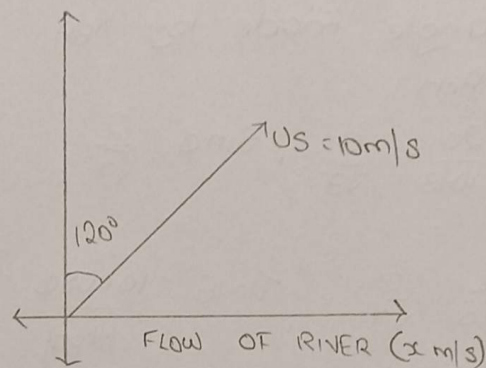


JEE QUESTIONS - MOTION IN A PLANE

1) A PERSON IS SWIMMING WITH A SPEED OF 10 m/s AT AN ANGLE OF 120° WITH THE FLOW AND REACHES TO A POINT DIRECTLY OPPOSITE ON THE OTHER SIDE OF THE RIVER. THE SPEED OF FLOW IS ' x ' m/s . THE VALUE OF ' x ' TO THE NEAREST INTEGER IS

Ans) Speed of swimmer relative to water (u_s) = 10 m/s
Speed of river = ' x ' m/s



$$\vec{u}_f = x\hat{i}$$

→ Swimmer's velocity relative to water = $10(\cos 120^\circ\hat{i} + \sin 120^\circ\hat{j})$

$$\text{We know } \cos 120^\circ = -\frac{1}{2} \text{ \& } \sin 120^\circ = \frac{\sqrt{3}}{2}$$

$$\text{So, } \vec{u}_s = 10\left(-\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) \\ = (-5\hat{i} + 5\sqrt{3}\hat{j})$$

$$\text{RESULTANT VELOCITY} = \vec{u}_s + \vec{u}_f \\ = (-5+x)\hat{i} + 5\sqrt{3}\hat{j}$$

⇒ FOR THE SWIMMER TO REACH DIRECTLY OPPOSITE, HORIZONTAL DRIFT MUST BE ZERO

$$-5+x=0$$

$$\boxed{x=5}$$

2. SHIP A IS 10KM DUE WEST OF SHIP B. SHIP A IS HEADING NORTH AT A SPEED OF 30 km/h, WHILE SHIP B IS HEADING IN A DIRECTION 60° WEST OF NORTH AT A SPEED OF 20km/h. FIND THEIR CLOSEST DISTANCE OF APPROACH AND THE TIME TAKEN FOR ~~THE~~ CLOSEST APPROACH

ANS. $\vec{v}_A = 30\hat{j}$, $\vec{v}_B = 20\sin 60^\circ(-\hat{i}) + 20\cos 60^\circ(\hat{j})$
 $\vec{v}_B = -10\sqrt{3}\hat{i} + 10\hat{j}$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -20\hat{j} - 10\sqrt{3}\hat{i}$$

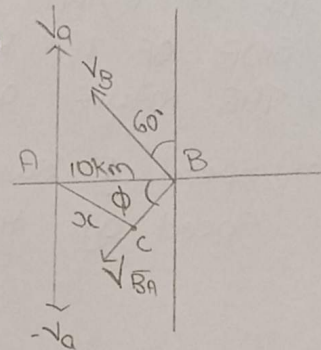
If ϕ is the angle made by $\vec{v}_{BA}$ with x axis, then

$$\tan \phi = \frac{20}{10\sqrt{3}} = \frac{2}{\sqrt{3}}, \quad \sin \phi = \frac{2}{\sqrt{7}}$$

FROM, $\triangle ABC$, $\frac{x}{10} = \frac{2}{\sqrt{7}}$

$$x = \frac{20}{\sqrt{7}}$$

TIME = $\frac{10 \cos \phi}{|\vec{v}_{BA}|} = \frac{10(\frac{\sqrt{3}}{\sqrt{7}})}{\sqrt{700}} = \frac{\sqrt{3}}{7} \text{ hr}$



3) A BALL IS THROWN FROM THE TOP OF A TOWER 61m HIGH, WITH VELOCITY 24.4m/s AT AN ~~ANGLE~~ ELEVATION OF 30° ABOVE THE HORIZONTAL. WHAT IS THE DISTANCE FROM THE FOOT OF TOWER TO THE POINT WHERE THE BALL HITS?

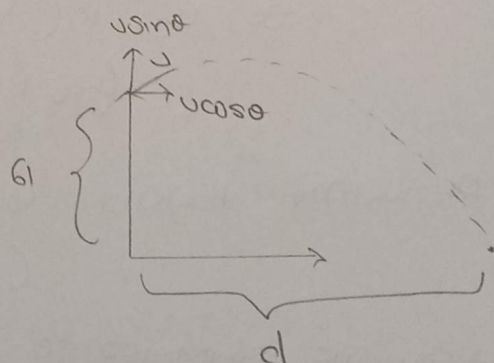
$$h = \frac{1}{2}gt^2 - ut_y$$

$$h = \frac{1}{2}gt^2 = u \sin \theta t$$

$$61 = 5t^2 - 4.2t$$

Solving, $t = \underline{5 \text{ sec}}$

Also, $d = (u \cos \theta)t = 105.65 \text{ m}$



Q The maximum speed of a boat in still water is 27 km/h . Now this boat is moving downstream in a river flowing at 9 km/h . A man in the boat throws a ball vertically upwards with speed of 10 m/s . Range of the ball as observed by an observer at rest on the bank is _____ cm
(take $g = 10 \text{ m/s}^2$) [JEE-main 2025]

Sol: To determine the horizontal velocity of the ball as seen by an observer on the bank

- The velocity of boat in still water is 27 km/h
 - Boat is moving downstream and the river flows at 9 km/h
- The actual speed of the boat relative to the bank is:
- $$v_{\text{boat}} = 27 + 9 = 36 \text{ km/h}$$

Convert km/h to m/s

$$v_{\text{boat}} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

↳ This is the horizontal velocity of the ball as seen by the observer on the bank

To calculate time taken by the ball to reach max height. Let the time to reach max height be t_{up}

$$t_{\text{up}} = \frac{u}{g} = \frac{10}{10} = 1 \text{ s}$$

Total time of flight $[T]$ is twice the time to reach max height

$$T = 2 \times 1 = 2 \text{ s}$$

The horizontal range R of the ball is the horizontal velocity multiplied by total time of flight

$$R = u \times T = 10 \times 2 = 20 \text{ m}$$

Convert this into cm

$$R = 20 \times 100 = \underline{2000 \text{ cm}}$$

Q Two balls with same mass and initial velocity, are projected at different angles in such a way that maximum height reached by first ball is 8 times higher than that of the second ball. T_1 and T_2 are the total flying times of first and second ball respectively, then the ratio of T_1 and T_2 is

Sol: Max height reached by first ball is 8 times that of the second ball

$$(H_{\max})_1 = 8 \times (H_{\max})_2$$

We can relate this to the initial velocities and angles using the formula for max height

$$\frac{u^2 \sin^2 \theta_1}{2g} = 8 \times \frac{u^2 \sin^2 \theta_2}{2g}$$

On simplifying

$$\sin \theta_1 = \sqrt{8} \sin \theta_2$$

$$\sin \theta_1 = 2\sqrt{2} \sin \theta_2$$

Now the ratio of the total flying times T_1 and T_2 is given by

$$\frac{T_1}{T_2} = \frac{2u \sin \theta_1 / g}{2u \sin \theta_2 / g} = \frac{\sin \theta_1}{\sin \theta_2} = 2\sqrt{2}$$

Thus the ratio of T_1 to T_2 is $2\sqrt{2} : 1$

Q A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in meters is _____.

[JEE-advance 2018]

$$\text{Sol: } H = \frac{u^2 \sin^2(45^\circ)}{2g} = 120 \text{ m}$$

$$\frac{u^2}{4g} = 120 \text{ m}$$

If speed is v after the first collision, then speed should remain $\frac{1}{\sqrt{2}}$ times, because kinetic energy has reduced to half

$$v = \frac{u}{\sqrt{2}}$$

$$h_{\max} = \frac{u^2 \sin^2(30)}{2g}$$

$$h_{\max} = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2(30)}{2g}$$

$$h_{\max} = \left(\frac{u^2/4g}{4}\right) = \frac{120}{4}$$

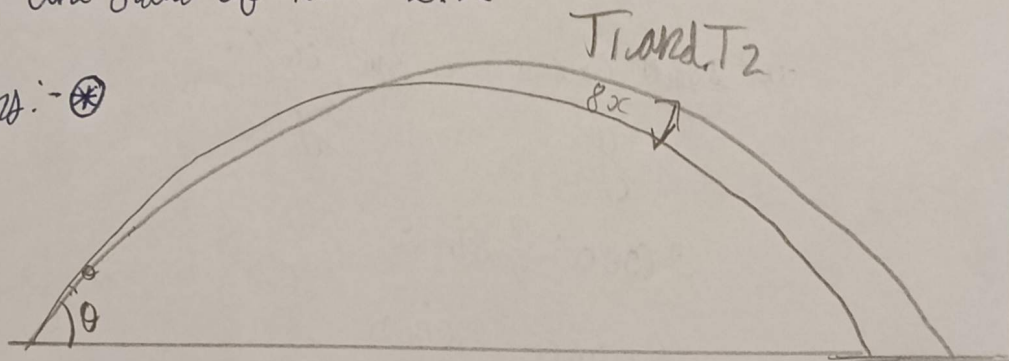
$$h_{\max} = 30 \text{ m}$$

Motion in A Plane

①

Q. 1) Two balls with same mass and initial velocity, were projected at different angles in such a way that $H_{\max}$ reached by first ball is 8 times higher than that of the second ball. T_1 and T_2 are the total flying times of first and second ball, respectively, then the ratio of T_1 and T_2 . [JEE Mains 2025]

Ans: - *



$$(H_1)_{\max}^2 = (H_2)_{\max} \times 8$$

$$\frac{u^2 \sin^2 \theta_1}{2g} = \frac{8 \times u^2 \sin^2 \theta_2}{2g}$$

$$\frac{\sin^2 \theta_1}{\sin^2 \theta_2} = \frac{8}{1}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{2\sqrt{2}}{1}$$

$$\frac{T_1}{T_2} \Rightarrow * \sin \theta_1 : \sin \theta_2 \Rightarrow \underline{\underline{2\sqrt{2}:1}}$$

Q.2) A particle is projected with velocity u so that its horizontal range is three times the $H_{\max}$ attained by it. The horizontal range of the projectile is given as $\frac{nu^2}{2g}$. What is the value of n . [JEE Mains 2025] (2)

Ans. $\textcircled{*}$ $R = \frac{u^2 \sin 2\theta}{g}$; $H_{\max} = \frac{u^2 \sin^2 \theta}{2g}$

Given:-

$$R = 3 H_{\max}$$

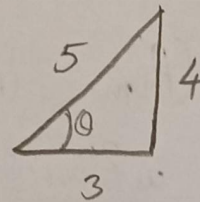
$$\frac{u^2 \sin 2\theta \cdot \cos \theta}{g} = \frac{3u^2 \sin^2 \theta}{2g}$$

$$2 \cos \theta = \frac{3}{2} \sin \theta$$

$$4 \cos \theta = 3 \sin \theta$$

$$\tan \theta = \frac{4}{3}$$

$$\therefore \theta = 53^\circ$$



$$R = \frac{u^2 \cdot 2 \cdot \frac{3}{5} \cdot \frac{4}{3}}{g}$$

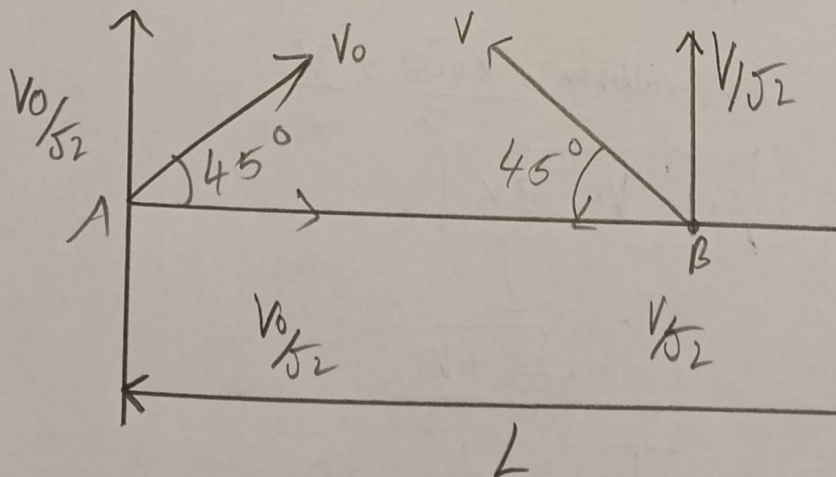
$$R = \frac{24u^2}{2g} = \frac{nu^2}{2g}$$

$$\therefore n = 24$$

Q.3) A ball is thrown from the location $(x_0, y_0) = (0, 0)$ of a horizontal playground with an initial speed v_0 at an angle θ_0 from the $+x$ -direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown at angle $(180 - \theta_1)$ from the $+x$ -direction with a suitable initial speed. For a fixed v_0 , when $(\theta_0, \theta_1) = (45^\circ, 45^\circ)$, the stone hits the ball after sometime, T_1 , and when $(\theta_0, \theta_1) = (60^\circ, 30^\circ)$, it hits the ball after time T_2 . In such case $(T_1/T_2)^2$ is: [JEE Advanced 2024]

Ans: (*) Let's take two cases:-

Case I:-



$$a_{\text{rel}} = 0$$

$$\text{For collision, } \frac{v_0}{\sqrt{2}} = \frac{V}{\sqrt{2}}$$

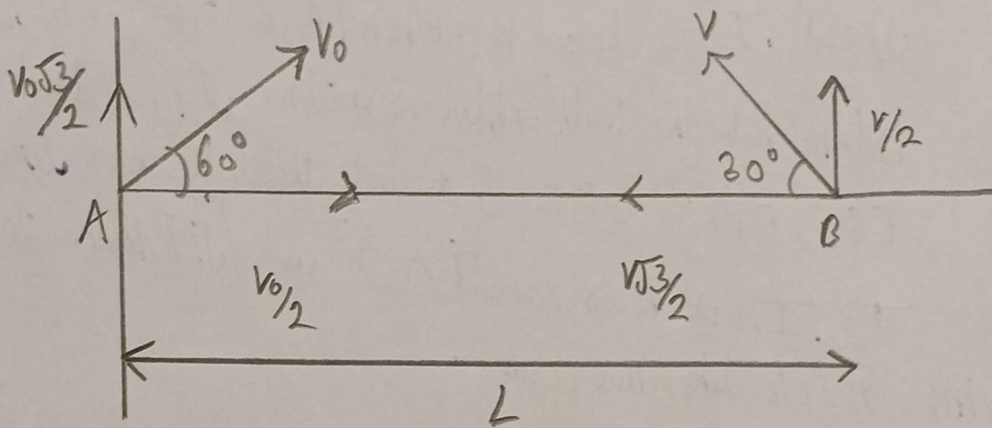
$$\boxed{V = v_0}$$

$$\text{So, } T_1 = \frac{L}{\frac{v_0}{\sqrt{2}} + \frac{V}{\sqrt{2}}}$$

(4)

$$\therefore T_1 = \frac{L}{\sqrt{2} v_0} \quad \text{--- (1)}$$

Case :- II



$$a_{\text{rel}} = 0$$

$$\text{For collision, } \frac{v_0 \sqrt{3}}{2} = \frac{V}{2}$$

$$\therefore V = \sqrt{3} v_0$$

$$T_2 = \frac{L}{\frac{v_0}{2} + \frac{V \sqrt{3}}{2}}$$

$$T_2 = \frac{L}{2 v_0} \quad \text{--- (2)}$$

So, from (1) and (2);

$$\therefore \text{Ans: } \left(\frac{T_1}{T_2} \right)^2 = \left(\frac{\frac{L}{\sqrt{2} v_0}}{\frac{L}{2 v_0}} \right)^2 = \left(\frac{2 v_0}{\sqrt{2} v_0} \right)^2 = \left(\frac{2}{\sqrt{2}} \right)^2 = 2$$

Motion In A Plane

TFE Questions

Q1. A particle is projected from the ground with an initial velocity of 40 m/s at an angle of 60° above the horizontal. Find the maximum height reached by the particle and the time taken to reach the maximum height.

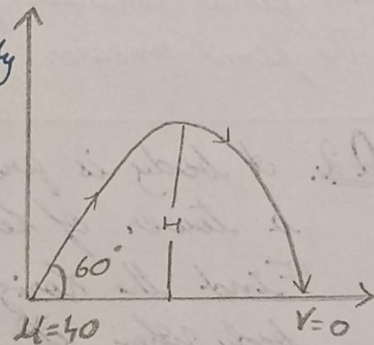
Ans. The maximum height 'H' is given by

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

where, u = initial velocity
 $= 40 \text{ m/s}$

θ = angle of projection
 $= 60^\circ$

g = acceleration due to gravity
 $= 10 \text{ m/s}^2$



$$\begin{aligned} \therefore H &= \frac{(40)^2 \sin^2 60^\circ}{2 \times 10} \\ &= \frac{1600 \times \frac{3}{4}}{20} \\ &= \frac{1200}{20} \end{aligned}$$

$$\therefore H = \underline{\underline{60 \text{ meters}}}$$

The time taken 't' is given by

$$t = \frac{u \sin \theta}{g}$$

$$\therefore t = \frac{40 \times \sin 60^\circ}{10}$$

$$= \frac{40 \times \sqrt{3}/2}{10}$$

$$\therefore t = 2\sqrt{3} \text{ seconds}$$

The final answer for maximum height = 60 m

The final answer for time taken = $2\sqrt{3}$ seconds

Q.2. A body is projected horizontally from the top of a tower of height 80 m with a velocity of 20 m/s. Find the horizontal distance covered by the body when it hits the ground.

Ans. Using equation $s = ut + \frac{1}{2}at^2$

where, $s = 80$

$u = 0$ (in this case)

$a = 10$

$$\therefore 80 = 0 + \frac{1}{2} \times 10 \times t^2$$

$$\therefore t^2 = 16$$

$$\therefore t = 4 \text{ seconds}$$

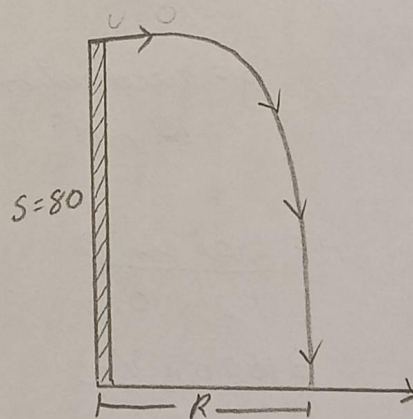
The horizontal distance covered is given by

$R = u \times t$ where $u = \text{horizontal velocity} = 20 \text{ m/s}$

$$\therefore R = 20 \times 4$$

$$\therefore R = 80 \text{ meters}$$

The final answer = 80 meters

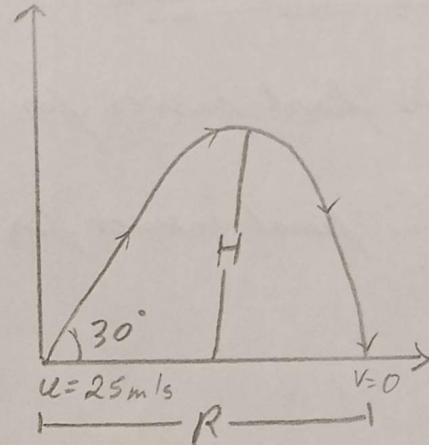


Q 3. A body is projected with an initial velocity of 25 m/s at an angle of 30° above the horizontal. Find the maximum height reached by the body and the horizontal distance covered when it reaches the maximum height.

Ans. The maximum height 'H' is given by,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

where $u = 25 \text{ m/s}$
 $g = 10 \text{ m/s}^2$



$$\therefore H = \frac{(25)^2 \sin^2 30^\circ}{2 \times 10}$$

$$= \frac{625 \times \frac{1}{4}}{20}$$

$$\therefore H = \frac{625}{80}$$

$$\therefore H = \underline{\underline{7.8125 \text{ meters}}}$$

The time taken 't' is given by $t = \frac{u \sin \theta}{g}$

$$t = \frac{25 \times \sin 30^\circ}{10}$$

$$= \frac{25 \times \frac{1}{2}}{10}$$

$$\therefore t = \underline{\underline{1.25 \text{ seconds}}}$$

The horizontal distance covered is given by

$$R = u \cos \theta \times t$$

$$R = 25 \cos 30^\circ \times 1.25$$

$$= 25 \times \frac{\sqrt{3}}{2} \times 1.25$$

$$\therefore \underline{R = 27.06 \text{ meters}}$$

The final answer for maximum height = 7.8125 m

The final answer for horizontal distance = 27.06 m

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2023-2024

Q.1 2 balls with same mass and initial velocity, are projected at different angles in such a way that max. height reached by first ball is 8 times higher than that of the 2nd ball. T_1 & T_2 are the total flying times of 1st & 2nd balls respectively, then the ratio of T_1 & T_2 is

→ Given, $(H_{\max})_1 = 8(H_{\max})_2$

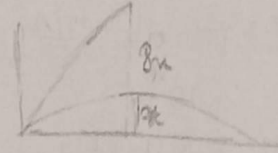
$$\therefore \frac{u^2 \sin^2 \theta_1}{2g} = 8 \left(\frac{u^2 \sin^2 \theta_2}{2g} \right)$$

$$\Rightarrow \sin \theta_1 = 2\sqrt{2} \sin \theta_2$$

Now, ratio of the total flying times T_1 & T_2 :-

$$\frac{T_1}{T_2} = \frac{2u \sin \theta_1 / g}{2u \sin \theta_2 / g} = \frac{\sin \theta_1}{\sin \theta_2} = \underline{\underline{2\sqrt{2} : 1}}$$

$$\therefore \underline{\underline{T_1 : T_2 = 2\sqrt{2} : 1}}$$



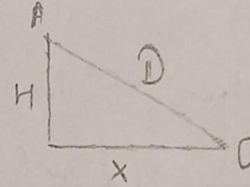
Q.2. A helicopter flying horizontally with a speed of 360 km/h at an altitude of 2 km , drops an object at an instant. The object hits the ground at a point O , 20 s after it's dropped. Displacement of O from the position of helicopter where the object was released is:
($g = 10 \text{ m/s}^2$)

$$\rightarrow u = 360 \times \frac{5}{18} = \underline{100 \text{ m/s}}$$

$$x = ut = 2 \times 10^3 \text{ m}$$

$$t = \sqrt{\frac{2H}{g}} \Rightarrow H = \frac{t^2 g}{2} = \frac{400 \times 10}{2} = \underline{2000 \text{ m}}$$

$$D = \sqrt{x^2 + H^2} \Rightarrow \underline{D = 2\sqrt{2} \text{ km}}$$



Q.3.

A particle is projected with velocity u so that its horizontally horizontal range is three times the max. height attained by it. The horizontal range of the projectile is given as $\frac{nu^2}{25g}$, where the value of 'n' is:

$$\rightarrow \text{Range } R = \frac{u^2 \sin 2\theta}{g}$$

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Given, } R = 3H_{\max}$$

$$\therefore \frac{u^2 \sin 2\theta}{2g} = 3 \cdot \frac{u^2 \sin^2 \theta}{2g}$$

$$2 \sin \theta \cos \theta = \frac{3}{2} \sin^2 \theta$$

$$\Rightarrow \sin 2\theta = \frac{3}{2} \sin^2 \theta$$

Dividing both sides by $\sin \theta$,

$$2 \cos \theta = \frac{3}{2} \sin \theta$$

$$\Rightarrow \tan \theta = \frac{4}{3}$$

$$\theta = 53^\circ$$

$$\Rightarrow R = \frac{u^2 2\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)}{g}$$

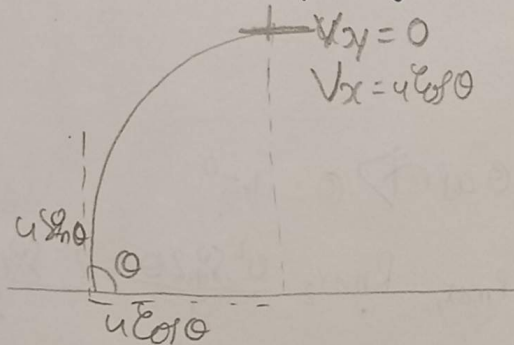
$$\Rightarrow R = \frac{24u^2}{25g}$$

$$\therefore n = 24$$

MOTION IN PLANE

Q) Initial speed of projectile fired from ground is 'u'. At highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$. The time of flight of projectile is:

Sol.



Given:

u = initial speed
 $V = V_x = u \cos \theta$ (at highest point)
 $= \frac{\sqrt{3}}{2}u$ ($V_y = 0$)

~~to given~~

$$V = V_x = u \cos \theta = \frac{\sqrt{3}}{2}u$$

$$\cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

$$\text{Time of flight} = \frac{2u \sin \theta}{g}$$

$$\text{TOF} = \frac{2 \times u \sin 30^\circ}{g}$$

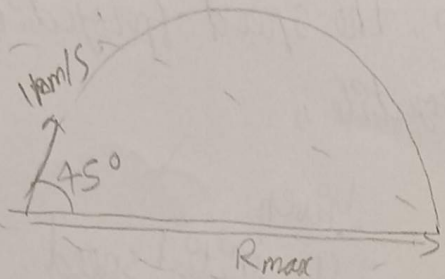
$$\text{TOF} = \frac{2 \times u \times \frac{1}{2}}{g} = \boxed{\frac{u}{g}}$$

Q) Two guns A and B can fire bullets at speed 1 km/s and 2 km/s respectively. From a point on horizontal ground they are fired in all possible directions. The ratio of maximum areas covered by bullets fired by 2 guns on ground is:

Sol.

① For gun A

$$u = 1 \text{ km/s}$$



To get $R_{\max}$, we have to take θ as $\theta = 45^\circ$

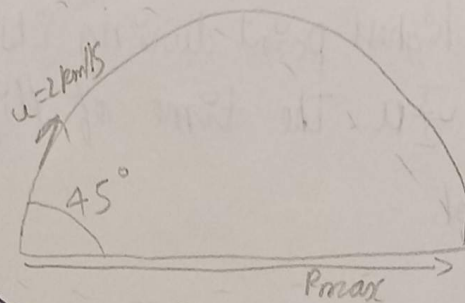
$$R_1 = \frac{u^2 \sin 2\theta}{g} = \frac{1^2 \sin 90}{g} = \frac{1}{g} = R_{\max_1}$$

$$A_{\max_1} = \pi R_{\max_1}^2 = \pi \times \frac{1}{g^2} = \frac{\pi}{g^2}$$

(as firing all direction forms)

② For gun B

$$u = 2 \text{ km/s}$$



$$R_{\max_2} = \frac{u^2 \sin 2\theta}{g} = \frac{2^2 \sin 90}{g} = \frac{4}{g}$$

$$A_{\max_2} = \pi R_{\max_2}^2 = \pi \times \left(\frac{4}{g}\right)^2 = \frac{16\pi}{g^2}$$

$$\frac{A_{\max_1}}{A_{\max_2}} = \frac{\frac{\pi}{g^2}}{\frac{16\pi}{g^2}} = \frac{1}{16}$$

$$\text{Ratio} \rightarrow 1:16$$

➤ Projectile projected with velocity of 25 m/s at θ with x axis. After 't' seconds its inclination with x axis becomes zero. ($R \rightarrow$ Horizontal Range). Value of θ will be:

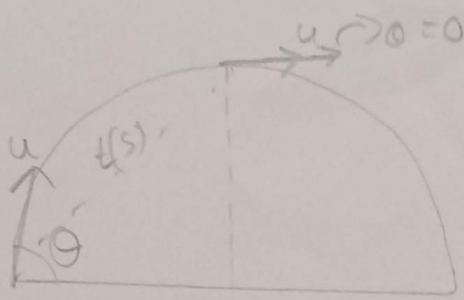
(A) $\frac{1}{2} \sin^{-1} \left(\frac{gt^2}{4R} \right)$

(B) $\frac{1}{2} \sin^{-1} \left(\frac{4R}{5t^2} \right)$

(C) $\tan^{-1} \left(\frac{4t^2}{5R} \right)$

(D) $\cot^{-1} \left(\frac{R}{20t^2} \right)$

Sol.



Given,

$u \rightarrow$ initial velocity
 $\theta \rightarrow$ angle of inclination with x axis

at $\theta = 0$, the body is at highest point
 So, $t(s)$ are req. to go to high point
 $u = 25 \text{ m/s}$, $g = 10 \text{ m/s}^2$

$$t = \frac{u \sin \theta}{g} = \frac{25 \sin \theta}{10} = \frac{5}{2} \sin \theta$$

(squaring t as in options only ' t^2 ' given)

$$t^2 = \frac{25}{4} \sin^2 \theta$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{625}{10} \times 2 \sin \theta \times \cos \theta = \frac{25 \times 5 \times \sin \theta \times 2 \cos \theta}{2}$$

(multiply & divide by $\frac{\sin \theta}{2}$, so that we can substitute t^2 in R)

$$R = \frac{25 \times 5}{2} \times \sin \theta \times 2 \cos \theta \times \frac{\sin \theta}{\frac{\sin \theta}{2}} = \frac{25 \sin^2 \theta}{4} \times \frac{2 \cos \theta}{\frac{\sin \theta}{2}} \times 5$$

$$R = t^2 \times \frac{2 \cos \theta}{\frac{\sin \theta}{2}} \times \frac{2}{\sin \theta} \times 5 = R = 20 t^2 \times \cos \theta \Rightarrow \cos \theta = \frac{R}{20 t^2}$$

$$\theta = \cos^{-1} \left(\frac{R}{20 t^2} \right) \quad \boxed{D}$$

Motion in a Plane

Q1) Two balls with same mass and initial velocity are projected at different angles in such a way that max. height reached by first ball is 8 times higher than that of the 2nd ball. T_1 and T_2 are the total flying times of first and second ball, respectively then the ratio of T_1 and T_2 is:

- (A) 2:1 (B) $\sqrt{2}:1$ (C) $2\sqrt{2}:1$ (D) 4:1

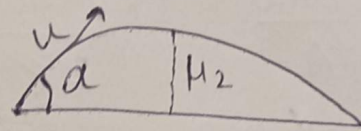
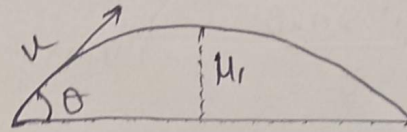
Ans:-

$u \rightarrow$ Initial velocity

$\theta \rightarrow$ Angle 1, $\alpha \rightarrow$ Angle 2

$g \rightarrow$ Gravitational Constant.

$$H_1 = 8 H_2$$



$$\Rightarrow \frac{u^2 \sin^2 \theta}{g} = \frac{u^2 \sin^2 \alpha}{g} \times 8$$

$$\Rightarrow \sin^2 \theta = \sin^2 \alpha \times 8$$

$$\Rightarrow \sin \theta = \sin \alpha \times 2\sqrt{2}$$

$$\Rightarrow \sin \theta = 2\sqrt{2} \sin \alpha$$

$$\therefore T_1 = \frac{2u \sin \theta}{g}; T_2 = \frac{2u \sin \alpha}{g}$$

$$\therefore \frac{T_1}{T_2} = \frac{2u \sin \theta}{g} \cdot \frac{g}{2u \sin \alpha}$$

$$\Rightarrow T_1 : T_2 = 2\sqrt{2} : 1 \quad [\text{C}]$$

Q2) Two projectiles are fired from ground with same initial speeds from same point at angles $(45^\circ + \alpha)$ and $(45^\circ - \alpha)$ with horizontal direction. The ratio of their times of flights is,

- (A) $\frac{1 + \tan \alpha}{1 - \tan \alpha}$ (B) $\frac{1 + \sin 2\alpha}{1 - \sin 2\alpha}$ (C) $\frac{1 - \tan \alpha}{1 + \tan \alpha}$ (D) 1

Ans:-

$$\theta_1 = 45^\circ + \alpha \quad \theta_2 = 45^\circ - \alpha; T = \frac{2u \sin \theta}{g}$$

$$\therefore \frac{T_1}{T_2} = \frac{\sin(45^\circ + \alpha)}{\sin(45^\circ - \alpha)} = \frac{\frac{1}{\sqrt{2}}(\cos \alpha + \sin \alpha)}{\frac{1}{\sqrt{2}}(\cos \alpha - \sin \alpha)} = \frac{1 + \tan \alpha}{1 - \tan \alpha} \quad [\text{A}]$$

Q3) A particle is projected with velocity u so that its horizontal range is three times the max. height attained by it. The horizontal range of the projectile is given as $\frac{nu^2}{25g}$, where value of n is:

[Given 'g' is the acceleration due to gravity.]

- (A) 6 (B) 12 (C) 18 ☒ (D) 24

Ans:-

$$R = \frac{u^2 \sin 2\theta}{g} \quad ; \quad H_{\max} = \frac{u^2 \sin^2 \theta}{g} \quad ; \quad R = 3H_{\max}$$

$$\therefore \frac{u^2 \sin 2\theta}{g} = 3 \times \frac{u^2 \sin^2 \theta}{g}$$

$$\Rightarrow 2 \sin \theta \cos \theta = \frac{3 \sin^2 \theta}{1} \quad \Rightarrow \tan \theta = \frac{4}{3}$$

$$\therefore R = \frac{u^2 \times 2 \times \frac{3}{5} \times \frac{4}{5}}{g} \quad \Rightarrow R = \frac{24 u^2}{25g} \quad \Rightarrow n = 24$$

MOTION IN A PLANE

1. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be

(a) $\frac{1}{2} \sin^{-1} \left[\frac{5t^2}{4R} \right]$ (b) $\frac{1}{2} \sin^{-1} \left[\frac{4R}{5t^2} \right]$ [use $g = 10 \text{ m/s}^2$]
 (c) $\tan^{-1} \left[\frac{4t^2}{5R} \right]$ (d) $\cot^{-1} \left[\frac{R}{2gt^2} \right]$

Solution:-

$$u \sin \theta = gt$$

$$25 \sin \theta = 10t \Rightarrow \sin \theta = \frac{2t}{5}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{625 \cdot 2 \sin \theta \cos \theta}{10} = 125 \sin \theta \cos \theta$$

substituting $\sin \theta = \frac{2t}{5}$

$$R = 125 \cdot \frac{2t}{5} \cdot \cos \theta = 50t \cos \theta \Rightarrow \cos \theta = \frac{R}{50t}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{2t}{5}}{\frac{R}{50t}} = \frac{2t}{5} \cdot \frac{50t}{R} = \frac{100t^2}{5R} = \frac{20t^2}{R}$$

$$\theta = \tan^{-1} \left(\frac{20t^2}{R} \right)$$

$$\theta = \cot^{-1} \left(\frac{R}{20t^2} \right)$$

$$(d) \theta = \cot^{-1} \left(\frac{R}{2gt^2} \right)$$

Ans:- option (d)

2. A girl standing on road holds a umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2} \text{ km}^{-1}$ the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is

- (a) 30 kmh^{-1} (b) $\frac{25}{\sqrt{2}} \text{ kmh}^{-1}$ (c) $\frac{30}{\sqrt{2}} \text{ kmh}^{-1}$ (d) 25 kmh^{-1}

Solution:-

horizontal - u
vertical - v

$$\tan 45^\circ = \frac{u}{v} = 1 \quad u = v$$

$$\text{Let } v_g = 15\sqrt{2} \text{ km/h}$$

$$u - v_g = 0 \Rightarrow u = v_g = 15\sqrt{2}$$

Since $u = v$, we have

$$v = 15\sqrt{2}$$

$$\text{speed} = v = 15\sqrt{2}$$

$$\text{Ans:- } 15\sqrt{2} = \frac{30}{\sqrt{2}} \quad (c)$$

3. A ball is projected with kinetic energy E , at an angle of 60° to the horizontal. The kinetic energy of the ball at the highest point of its flight will become

- (a) zero (b) $\frac{E}{2}$ (c) $\frac{E}{4}$ (d) E

$$\text{Solution:- } E = \frac{1}{2} m v^2 \quad \theta = 60^\circ$$

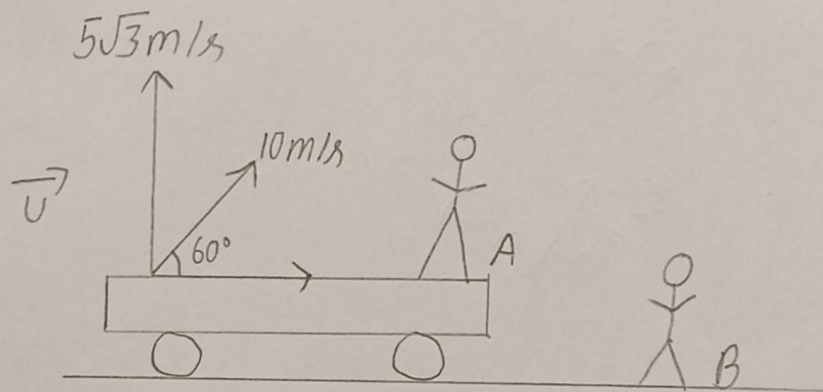
$$u_x = u \cos \theta$$

$$\cos 60^\circ = \frac{1}{2} \rightarrow v_x = \frac{u}{2}$$

$$E_{th} = \frac{1}{2} m \left(\frac{u}{2} \right)^2 = \frac{1}{2} m \cdot \frac{u^2}{4} = \frac{1}{4} \cdot \frac{1}{2} m u^2 = \frac{E}{4}$$

$$\text{Ans:- } \frac{E}{4} \quad (c)$$

Q.1 A train is moving along straight lines with a constant acceleration 'a'. A girl standing in the train throws a ball forward with a speed of 10 m/s , at an angle of 60° to the horizontal. The girl has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train is?



$$R = \frac{1}{2} at^2 + 1.15$$

$$U^2 \sin^2 \theta = \frac{1}{2} a \left(\frac{2U \sin \theta}{g} \right)^2 + 1.15$$

$$\frac{100 \times \sqrt{3}}{2} = \frac{1}{2} a \left(\frac{4 \times 100 \times \frac{3}{4}}{100} \right) + 1.15$$

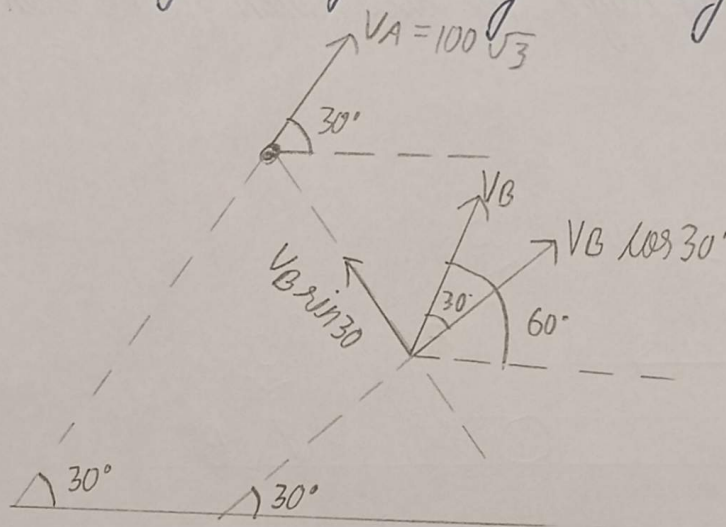
$$2 \times 5\sqrt{3} = 3a + 2.30$$

$$10\sqrt{3} - 2.30 = 3a$$

$$17.3 - 2.3 = 3a$$

$$\frac{15}{3} = 5 \text{ m/s}^2$$

Q.2 Airplanes A and B are flying with constant velocity in the same vertical planes at angles 30° and 60° with respect to the horizontal resp. as shown in the fig. The speed of A is $100\sqrt{3} \text{ m/s}$. At time, $t=0 \text{ s}$, an observer in A finds B at a distance of 500 m . If at $t=t_0$, A just escapes being hit by B, t_0 in sec is



$$V_A = V_B \cos 30$$

$$100\sqrt{3} = V_B \times \left(\frac{\sqrt{3}}{2} \right)$$

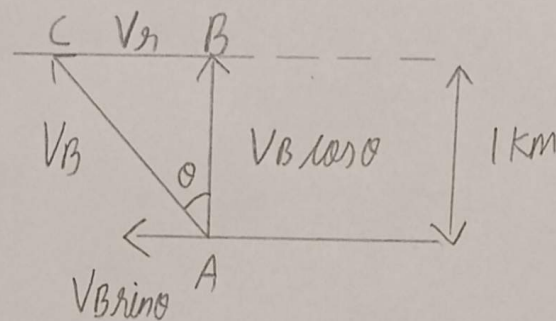
$$V_B = 200 \text{ m/s}$$

$$t_0 = \frac{d}{V_B \sin 30}$$

$$\frac{500}{200 \times \frac{1}{2}} = 5 \text{ s}$$

$$200 \times \frac{1}{2}$$

Q.3 A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river in the shortest time. The velocity of river water in km/hr



$$\text{Time } t = 15 \text{ min} = \frac{15}{60} = \frac{1}{4} \text{ hour}$$

$$\text{Velocity} = \frac{\text{displacement}}{\text{Time}}$$

$$V_B \cos \theta = \frac{1}{\frac{1}{4}} = 4 \text{ km/hr}$$

$$5 \cos \theta = 4 \text{ km/hr}$$

$$\cos \theta = \frac{4}{5}$$

$$\sin \theta = \frac{3}{5}$$

$$\text{Consider } \triangle ABC \quad \frac{V_r}{V_B} = \frac{3}{5}$$

$$V_r = 3 \text{ km/hr}$$

Motion in a plane

Q1. Position vector of a moving body

$$\vec{r} = (5t^2\hat{i} - 5t\hat{j}) \text{ m}$$

Find magnitude and directions of its velocity, $t = 2\text{ s}$

Sol:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} (5t^2\hat{i} - 5t\hat{j}) = 10t\hat{i} - 5\hat{j}$$

$$\vec{v} = 10 \cdot 2\hat{i} - 5\hat{j} = 20\hat{i} - 5\hat{j} \text{ (m/s)}$$

$$\text{magnitude} = \sqrt{20^2 + (-5)^2} = \sqrt{400 + 25} = \sqrt{425}$$

$$\Rightarrow 5\sqrt{17}$$

$$\tan \theta = \frac{v_y}{v_x} = \frac{-5}{20} = -\frac{1}{4}$$

$$\theta = 14.04^\circ \text{ below } "+x"$$

Q2. River flows from west to east at 9 km/h. Boat can move at 27 km/h in still water and steered at 150° . Boat crosses in $\frac{1}{2}$ minute. Find width of river.

Sol:

Boat speed $v = 27 \text{ km/h}$, angle $= 150^\circ$.

Perpendicular to river

$$\Rightarrow v \sin 150^\circ = 27 \cdot \sin 150^\circ \\ = 0.5$$

$$\Rightarrow 27 \times 0.5 = 13.5 \text{ km/h}$$

crossing time (h) $= 0.00833 \text{ h}$, width $= \frac{13.5}{120} \text{ km}$

$$\text{compute} = 13.5 / 120 = 135 / 1200 = 0.1125 \text{ km}$$

$$112.5 \text{ m}$$

Q3. 2 projectiles fired from ground with same "u"
at angles $(45^\circ + \alpha)$ $(45^\circ - \alpha)$. Horizontal range is
 $\frac{u^2}{25g}$. Find n .

Sol:

$$\text{at angle } \theta = R = \frac{u^2 \sin 2\theta}{g}$$

$$\text{for } \theta_1 = 45^\circ + \alpha : 2\theta_1 = 90^\circ + 2\alpha = \sin(90^\circ + 2\alpha) \\ \Rightarrow \cos 2\alpha$$

$$\text{for } \theta_2 = 45^\circ - \alpha : 2\theta_2 = 90^\circ - 2\alpha = \sin(90^\circ - 2\alpha) \\ \Rightarrow \cos 2\alpha$$

$$\text{each range} = R = \frac{u^2 \cos 2\alpha}{g}$$

$$R = \frac{u^2}{25g} \Rightarrow \cos 2\alpha = \frac{n}{25} = 25 \cos 2\alpha = n$$

$$25 \cos 2\alpha = n$$

- Q. A river is flowing from West to East direction with speed of 9 km h^{-1} . If a boat capable of moving at a max speed of 27 km h^{-1} in still water, crosses the river in half a minute, while moving with max speed at an angle of 150° to direction of river flow, then the width of river is:-

Sol: The $\perp$ to the river is:

$$27 \times \cos 60 \\ = 27/2 \text{ km/h}$$

Speed to ~~km/h~~ m/s.

$$27/2 \text{ km/h} = 27/2 \times 1000/3600 \\ = \frac{27}{2} \times \frac{5}{18} \text{ m/s}$$

∴ the boat takes 30 secs to cross:

dist = speed $\times$ time

$$s = \frac{27}{2} \times \frac{5}{18} \times 30 = \underline{\underline{112.5 \text{ m}}}$$

- Q. The max. vertical height to which a man can throw a ball is 136m. The max horizontal distance up to which he can throw the same ball is:-

Sol: $h = v^2 / 2g$

$$v = \sqrt{2gh} = \sqrt{2g \times 136} \quad \text{--- (i)}$$

For max range, $\theta = 45^\circ$

$$R_{\text{max}} = \frac{v^2}{g} \quad \text{--- (ii)}$$

(i) & (ii)

$$R_{\text{max}} = \frac{v^2}{g} = \frac{2g \times 136}{g} \\ = \underline{\underline{272 \text{ m}}}$$

Q. The initial speed of a projectile fired from ground is u . At the highest point during its motion the speed of projectile is $\frac{\sqrt{3}}{2} u$. The time of flight of the projectile is:

$$u \cos \theta = \frac{\sqrt{3}}{2} u$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$

$$\text{Time of flight} = \frac{2u \sin \theta}{g} = \left(\frac{u}{g} \right)$$

Q) Two projectiles are projected with same 'u' at 15° and 30° with respect to horizontal. The ratio of their ranges is $1:2$, The value of x

(A) $\sqrt{2}$

(C) $\sqrt{3}$

(B) $2\sqrt{3}$

(D) $\frac{1}{\sqrt{2}}$

Ans: $R = \frac{u^2 \sin 2\theta}{g}$

$\theta_1 = 15^\circ$

$R_1 = \frac{u^2}{2g}$

$\theta_2 = 30^\circ$

$R_2 = \frac{\sqrt{3}u^2}{2g}$

$\frac{R_1}{R_2} = \frac{1}{\sqrt{3}}$

$x = \sqrt{3}$

Q.) A gun mounted on the ground fires bullets in all directions with same speed. The farthest distance the bullets could reach is 6.4 m. The speed of the bullets from the gun is — m/s

Ans: $\therefore R_{\max} = \frac{u^2}{g}$

$$6.4 = \frac{u^2}{10}$$

$$u = 8 \text{ m s}^{-1}$$

Q.) If x and y coordinates of a projectile as a fcn of t are given as $24t$ and $43.6t - 4.9t^2$. Then the angle made by projectile with horizontal when $t=2\text{s}$ is —

(A) 60

(B) 45

(C) 30

(D) 75

Ans: $v_x = \frac{d}{dt} (24t) = 24$

$$v_y = \frac{d}{dt} (43.6t - 4.9t^2) = 43.6 - 9.8t$$

At $t=2\text{s}$,

$$v_x = 24$$

$$v_y = 24$$

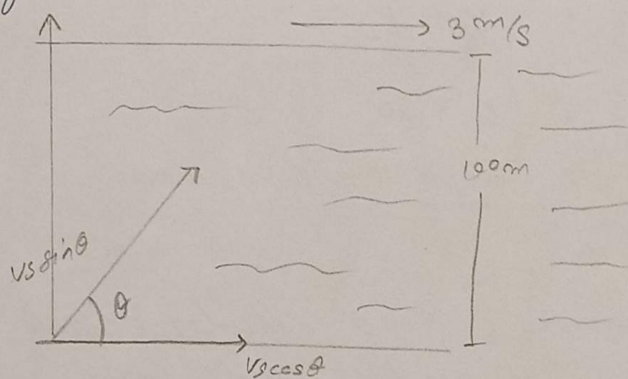
$$\theta = \frac{v_y}{v_x} = 1$$

$\theta = 45^\circ$

SAANVI PATEL, XI-A, (BATCH - (2025-2026))

Q. A river is 100m wide and flows downstream at $V_r = 3.0 \text{ m/s}$. A swimmer's speed relative to H_2O is $V_s = 5.0 \text{ m/s}$

- (i) What angle (wrt the line directly across the river, i.e. upstream from $\perp$) should the swimmer aim, so that she lands exactly opposite her starting point? Find time to cross
- (ii) If she instead swims straight across, how far downstream will she be carried.



a) To land opposite ; net x -velocity = 0

$$-V_s \sin \theta + V_r = 0$$

$$\sin \theta = \frac{V_r}{V_s} = \frac{3}{5} = 0.6$$

$$\boxed{\theta = 0.6 \sin^{-1}} \rightarrow \underline{\underline{Ans}}$$

Across component $\rightarrow V_s \cos \theta$

$$\cos \theta = \sqrt{1 - (0.6)^2} = \sqrt{0.64} = 0.8$$

$$\begin{aligned} V_{\text{across}} &= V_s \cos \theta \\ &= 0.5 \times 0.8 \\ &= \underline{\underline{4.0 \text{ m/s}}} \end{aligned}$$

⇒ Angular speed at $t = 2 \text{ sec}$.

$$\omega = \omega_0 + \alpha t$$

$$= 1 + 4(2)$$

$$\omega = \underline{9 \text{ rad/s}}$$

a) Radial Acceleration

$$a_r = r \omega^2$$

$$= 2(9)^2$$

$$= \underline{162 \text{ m/s}^2} \longrightarrow \underline{\text{Ans}}$$

b) Tangential Acceleration.

$$a_t = r \alpha$$

$$= 2(4)$$

$$= \underline{8 \text{ m/s}^2} \longrightarrow \underline{\text{Ans}}$$

c) Total acceleration :

$$a = \sqrt{a_r^2 + a_t^2}$$

$$= \sqrt{162^2 + 8^2}$$

$$= \sqrt{26308}$$

$$a = \underline{\underline{\text{approx } 162.2 \text{ m/s}^2}}$$

approx

→ Ans

d) Angle θ b/w, total acceleration and radial inward direction.

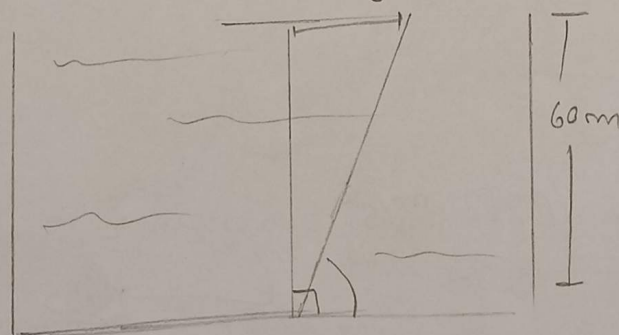
$$\tan \theta = \frac{a_t}{a_r} = \frac{8}{162} = 0.049$$

$$\theta = 0.049 \tan^{-1} \rightarrow \text{Ans}$$

Q. A river is 60 m and flows downstream at 4 m/s. A swimmer can swim at 6 m/s relative to the water and aim straight across ($\perp$ to banks)

Find :

- i) Time to cross
- ii) How far downstream she is carried
- iii) Magnitude and direction of her resultant velocity relative to ground.



⇒

(i) Time = ?

$$V_s = 6 \text{ m/s}$$

$$t = \frac{\text{width}}{\text{across speed}} = \frac{60}{6} = \underline{\underline{10 \text{ sec.}}}$$

↳ Ans

(ii) Downstream drift : $V_r = 4 \text{ m/s}$

$$= \text{river speed} \times t$$

$$= 4 \times 10$$

$$= \underline{\underline{40 \text{ m}}} \longrightarrow \underline{\underline{\text{Ans.}}}$$

(iii) Resultant ground velocity :

$$V_y = 6 \text{ m/s}$$

$$V_x = 4 \text{ m/s}$$

Magn :

$$V_R = \sqrt{6^2 + 4^2}$$

$$= \sqrt{52}$$

$$= \underline{\underline{7.21 \text{ m/s}}} \text{ approx}$$

↳ Ans

Direction :

$$\tan \theta = \frac{V_x}{V_y}$$

⇒

$$\tan \theta = \frac{4}{6}$$

$$\Rightarrow \theta = \tan^{-1} \frac{4}{6}$$

↳ Ans